Complexity Example *

Jens Lechtenbörger

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Sample Algorithms

- Determine Big O complexity for following algorithms in Python!
- Background
 - This presentation embeds klipse, to enable live code execution.
 - * Thus, click into code on next slides, edit it, and have results immediately displayed.
 - $\cdot\,$ If code does not execute, maybe reload without cache (Ctrl+F5 in Firefox)
 - $\cdot\,$ Based on in-browser implementation of Python (skulpt), not complete.

Instructions

- 1. Figure out what the algorithms on the next slides do.
 - If you are not sure, maybe copy&paste into Python Tutor, which enables step-by-step execution with visualizations of values.
- 2. Determine the algorithms' complexities in terms of numbers of necessary plus operations.
 - If you are puzzled about the focus on plus operations, note that they occur at the inner-most level of nesting in while loops. For each iteration of a loop, a fixed number of other operations is executed, and those are covered by a constant factor in the definition of Big O complexity (*M* at Wikipedia.)

Subsequent quizzes lead to solutions. Please try yourself first.

^{*}This PDF document is an inferior version of an OER in HTML format; free/libre Org mode source repository.

Naive Multiplication

```
def naive_mult(op1, op2):
    if op2 == 0: return 0
    result = op1
    while op2 > 1:
    result += op1
    op2 -= 1
    return result
```

```
print(naive_mult(2, 3))
```

- Some notes
 - Code on left is meant for non-negative integers
 - * Better code would test this
 - Python basics
 - * def naive_mult(op1, op2) declares function naive_mult with two operands
 - * == tests for equality, = is assignment to variable on left
 - * result += op1 is short
 for result = result
 + op1
 - thus, op1 is added to result
 - \cdot -= similarly
 - * return exits the function, delivers result

A solution

Naive Exponentiation

```
def naive_mult(op1, op2):
    if op2 == 0: return 0
    result = op1
    while op2 > 1:
    result += op1
    op2 -= 1
    return result
def naive_exp(op1, op2):
    if op2 == 0: return 1
    result = op1
    while op2 > 1:
    result = naive_mult(result, op1)
    op2 -= 1
    return result
```

```
print(naive_exp(2, 3))
```

- Some notes
 - naive_mult is copied from previous slide
 - naive_exp shares same basic structure
 - * But with invocation of naive_mult instead of plus operation

A solution

A "Small" Change

- What happens if the order of arguments to naive_mult on the previous slide was reversed, i.e., if naive_mult(op1, result) instead of naive_mult(result, op1) was executed?
 - Clearly, as multiplication is commutative, the result does not change.
 - What about the resulting complexity?

A surprise?

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